Electromagnetic Fields in Matter: Six Equations for Two Fields*

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A fresh analysis of the four Maxwell's equations shows that in the presence of matter, the behavior of electromagnetic fields is described by six equations (not only four). These equations describe two fields, only. The ideas presented in this text give an interpretation of classical electromagnetism that is made consistent with modern field theories like QED or QCD.

I. INTRODUCTION

Traditionally, electrical fields are separated into a "stimulating field", **D** (usually called electrical displacement) and and "polarization field", **P** . Magnetic phenomena are attributed to a "magnetizing filed" **H** (sometimes called "magnetic field") and a "magnetization field", **M** (see for example [1],[2])

Separating stimulating fields from the rest is a unique feature of classical electrodynamics. Neither in quantum electrodynamics (QED, see for example[3]), nor in the theory of general relativity, nor in any other widely accepted dynamic theory, such a separation is made. Also, it turns out to be an concept that cannot be verified.

In the following section, it will be shown that what has traditionally been assumed to be a separation between stimulation fields (**D** and **H**) and fields that can be detected (**E** and **B**) is the result of an attempt to separate field contributions from bound charges and free charges.

Using the four well-known Maxwell's equations together with the principle of linear superposition for the case in which matter is present, the whole of classical electromagnetism may be reproduced. Some field quantities, however, get new meanings. As a by-product, two more equations relating electromagnetic fields to bound charges and currents are derived.

II. MAXWELL'S EQUATIONS, APPLIED TO MATTER

The detection of a magnetic field is based on the Lorentz-force on a charge Q moving at a velocity **v** through this field represented by its flux density **B**: $\mathbf{F}_L = Q\mathbf{v} \times \mathbf{B}$. The detection of an electrical field **E** uses the Coulomb-force $\mathbf{F}_C = Q\mathbf{E}$.

Maxwell's equations may be now written in a form containing measurable field quantities, only:

$$\begin{array}{ll} \nabla \cdot \varepsilon_0 \mathbf{E} = \rho & \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} & \nabla \times (\mu_0^{-1}\mathbf{B}) = \mathbf{J} + \frac{\partial}{\partial t}\varepsilon_0 \mathbf{E} \, . \end{array} \tag{1}$$

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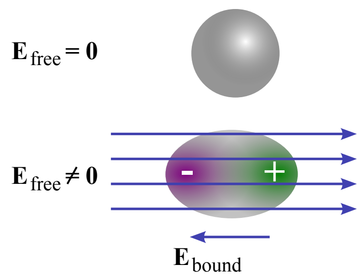

FIG. 1. Sketch of polarization according to [4]: an external field $\mathbf{E}_{\text{free}}$ leads to a separation of charges bound within an atom or molecule. These charges produce a field $\mathbf{E}_{\text{bound}}$.

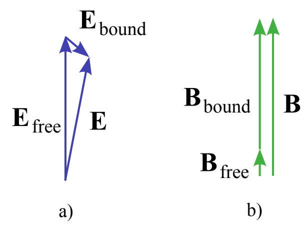

FIG. 2. In the presence of matter, the fields **E** and **B** are calculated as the vector sum ow two contributions (taken from [4]) On the left side, a), the case of an electrically anisotropic material is shown, for b), a ferromagnetic substance was assumed.

In (1), ρ is the charge density, **J** is the current density, ε_0 is the permittivity of free space, and μ_0 is the permeability of free space. Linearity of these equations guarantees the principle of linear superposition to apply to charge densities, current densities, and the fields **E** and **B**.

Linear superposition can be used to describe the modification of fields due to the presence of matter. Fig. 1 shows that an external field $\mathbf{E}_{\text{free}}$, applied to a neutral molecule will lead to the appearance of a new field, $\mathbf{E}_{\text{bound}}$. Polarization suggests to separate the charges into a set that has sufficient energy to move freely, and an energetically distinct set of charges which do not have enough energy to move away from the atoms (or molecules) they are bound to. Magnetization suggests a similar distinction for currents (see also Fig. 2):

$$\begin{array}{lll} \rho = \rho_{\text{free}} + \rho_{\text{bound}} & \text{and} & \mathbf{E} = \mathbf{E}_{\text{free}} + \mathbf{E}_{\text{bound}} \, , \\ \mathbf{J} = \mathbf{J}_{\text{free}} + \mathbf{J}_{\text{bound}} & \text{and} & \mathbf{B} = \mathbf{B}_{\text{free}} + \mathbf{B}_{\text{bound}} \, . \end{array} \tag{2}$$

Maxwell's equation will hold for either subset, for example

$$\begin{aligned} \nabla \cdot \varepsilon_0 \mathbf{E}_{\text{free}} &= \rho_{\text{free}} \\ \nabla \times (\mu_0^{-1} \mathbf{B}_{\text{free}}) &= \mathbf{J}_{\text{free}} + \tfrac{\partial}{\partial t} \varepsilon_0 \mathbf{E}_{\text{free}} \,. \end{aligned} \tag{3}$$

Inserting (2) into (3) gives

$$\begin{aligned} \nabla \cdot \varepsilon_0 (\mathbf{E} - \mathbf{E}_{\text{bound}}) &= \rho_{\text{free}} \\ \nabla \times [\mu_0^{-1} (\mathbf{B} - \mathbf{B}_{\text{bound}})] &= \mathbf{J}_{\text{free}} + \tfrac{\partial}{\partial t} \varepsilon_0 \mathbf{E}_{\text{free}} \,. \end{aligned} \tag{4}$$

One may be familiar with (4), because it is widely known as "Maxwell's equations in matter" in the form

$$\begin{aligned} \nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = \nabla \cdot \mathbf{D} &= \rho_{\text{free}} \\ \nabla \times (\mu_0^{-1} \mathbf{B} - \mathbf{M}) = \nabla \times \mathbf{H} &= \mathbf{J}_{\text{free}} + \tfrac{\partial}{\partial t} \mathbf{D} \,. \end{aligned} \tag{5}$$

The latter three equations are simultaneously true if, and only if, the following identities apply:

$$\begin{aligned} \mathbf{H} &= \mu_0^{-1} \, \mathbf{B}_{\text{free}} \\ \mathbf{M} &= \mu_0^{-1} \, \mathbf{B}_{\text{bound}} \\ \mathbf{D} &= \varepsilon_0 \, \mathbf{E}_{\text{free}} \\ \mathbf{P} &= -\varepsilon_0 \, \mathbf{E}_{\text{bound}} \,. \end{aligned} \tag{6}$$

At this point, it becomes clear, why none of the quantities appearing on the left side of (6) can be found in books on QED: At distances below the size of an electron's orbit, it is no longer appropriate to make a difference between free and bound electrons: Within a volume limited by such distances, even bound electrons may move freely.

Further aspects worth noticing are:

- Equation set (4) was derived from (1). The first set of equations therefor holds whether or not matter is present.
- Classical electromagnetism knows two measurable types of fields, only: **E** and **B**.
- As neither the Coulomb-force nor the Lorentz-force can distinguish between different sources of fields, only the sums $\mathbf{E} = \mathbf{E}_{\text{free}} + \mathbf{E}_{\text{bound}}$ and $\mathbf{B} = \mathbf{B}_{\text{free}} + \mathbf{B}_{\text{bound}}$ can be measured in the presence of matter.
- Therefor, **H** and **D** can only be measured under vacuum conditions, while **M** and **P** can only be measured in the absence of free currents and charges.

Hence, the idea of an **H** field traversing and stimulating matter can neither be verified nor falsified experimentally. A corresponding statement holds for **D**.

III. TWO MORE EQUATIONS

Equations (2) and (3) may also be solved for bound charges and currents. Hence, in total, one is left with six equations:

$$\begin{aligned} \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\tfrac{\partial}{\partial t} \mathbf{B} \\ \nabla \cdot \varepsilon_0 (\mathbf{E} - \mathbf{E}_{\text{bound}}) &= \rho_{\text{free}} \\ \nabla \times [\mu_0^{-1} (\mathbf{B} - \mathbf{B}_{\text{bound}})] &= \mathbf{J}_{\text{free}} + \tfrac{\partial}{\partial t} \varepsilon_0 \mathbf{E}_{\text{free}} \\ \nabla \cdot \varepsilon_0 (\mathbf{E} - \mathbf{E}_{\text{free}}) &= \rho_{\text{fbound}} \\ \nabla \times [\mu_0^{-1} (\mathbf{B} - \mathbf{B}_{\text{free}})] &= \mathbf{J}_{\text{bound}} + \tfrac{\partial}{\partial t} \varepsilon_0 \mathbf{E}_{\text{bound}} \end{aligned} \tag{7}$$

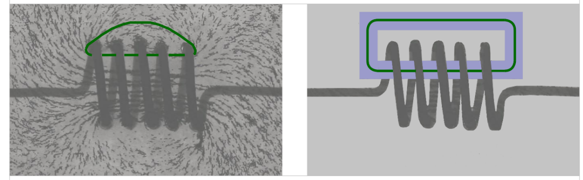

FIG. 3. Magnetic field lines near a coil, indicated by iron filings (a). On the right, the supposed lines inside a ferromagnetic core (b) are shown.

It may easily be verified that in the absence of a distinction between free and bound charges and currents, this set of equations reduces to Maxwell's equations as given in (1). The last two equations in (7) are the new ones. In traditional notation, they may be formulated in the manner

$$\begin{aligned} \nabla \cdot (\varepsilon_0 \mathbf{E} - \mathbf{D}) = -\nabla \cdot \mathbf{P} &= \rho_{\text{bound}} \\ \nabla \times (\mu_0^{-1} \mathbf{B} - \mathbf{H}) = \nabla \times \mathbf{M} &= \mathbf{J}_{\text{bound}} - \tfrac{\partial}{\partial t} \mathbf{P} \,, \end{aligned} \tag{8}$$

as can be obtained by inserting (6) into (7).

IV. TWO FIELDS, ONLY

The symbols **D**, **P**, **H** and **M** all represent contributions to the fields **B** and **E**. None of them is a "field" of its own kind, as may be deduced from the fact that drawing their "field lines" requires the knowledge of **B** and **E**. A simple example is shown in Fig. 3. The field lines shown on both sides of the figure are often attributed to "the H field". However, the shape an strength of the field shown on the right (Fig.3. b) requires knowledge of both, the current in the coil and the properties of the core.

In the past, an extensive use of the term "field" has been misleading by suggesting that there would be more than one type of magnetic field, and more than one type of electric field. Some textbook sections on Maxwell's equations [5] even start with a sentence like "Maxwell's equations are about the four fields E, D, H and M".

Others, [6], [4] model the effect of matter as a modification oft the fields from free charges and currents using tensor valued functions ε_r and μ_r^{-1}, defined by

$$\mathbf{E}_{\text{free}} = \varepsilon_r \mathbf{E} \quad \text{and} \quad \mathbf{B}_{\text{free}} = \mu_r^{-1} \mathbf{B} \,. \tag{9}$$

In this manner, there is also no more need for fields, other than **E** and **B**. This paper supports the point of view taken by these books by coming to a consistent conclusion via different reasoning.

V. CONCLUSION

The fields **E** and **B** are defined by their effects on charges as given by the electrodynamic forces. Contributions to these fields based on a distinction between free and bound charges are **D**, **P**, **H** and **M** with definitions based on (6). In this paper, the relation between these contributions and bound charges and currents is presented in addition to their well-known relations to free currents and charges.

The idea of **D** and **H** "fields" traversing and stimulating matter may be skipped, as it can neither be verified nor falsified experimentally. In this manner, classical electromagnetism can be relieved from a property distinguishing it from all other widely accepted dynamic theories.

[1] W. Panaofsky and M. Philips, *Classical Electricity and Magnetism* (Addison, 1962).

[2] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, 1998).

[3] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, 1965).

[4] M. Poppe, *Die Maxwellsche Theorie* (Springer, 2015).

[5] ..., *intentionally left blank* (...).

[6] P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers* (Palgrave Macmillan, 2003).